Rent-to-Own Usurers? Theory and Empirical Evidence

Sanjiv Jaggia *
Orfalea College of Business, California Polytechnic State University
Hervé Roche*
Universidad Adolfo Ibáñez

April 5, 2016

Abstract

We develop a rational-expectations competitive equilibrium model to explore the pricing mechanism of a rent-to-own agreement. It accounts for the agreement's unique features such as the return or early purchase options and several free services included in the contract. Using detailed transactional data, we infer how customers exercise these options to calibrate our model for several product categories, contractual lengths and payment periodicity. In all cases, our model is able to match the patterns displayed by the observed markups and APRs very well; however, it falls a little short in matching the levels.

JEL classification: D1, G23

Keywords: Embedded Options; Financial Distress; Subprime Industry.

*We are indebted to Michael H. Anderson for the data and for his meticulous and generous input. We also thank David Card, Jesus Sierra, Satish Thosar, Stathis Tompaidis, Eduardo Zambrano, Cal Poly, Adolfo Ibáñez University and Diego Portales University seminar participants for helpful comments and suggestions. All errors remain ours.

Corresponding author: Sanjiv Jaggia, Professor of Economics, Orfalea College of Business, California Polytechnic State University, San Luis Obispo, CA 93407, Tel: 805-756-7519, E-mail: sjaggia@calpoly.edu
Hervé Roche, Associate Professor of Finance, Universidad Adolfo Ibáñez, Avenida Diagonal las Torres 2640, Peñalolén, Santiago, Chile, Tel: +56 023311462, E-mail: herve.roche@uai.cl

1 Introduction

There have always been poor people and there have always been those who would lend to them. Pawnshops, for example, go back to ancient Greek and Roman times in the western world, and go even further back in China. At the same time, the subprime industry as we think of it today largely dates back to the 1970s. Starting in the mid-70s and continuing well into the 80s, the combination of high interest rates and state usury ceilings greatly impaired the ability of banks and other financial intermediaries to offer consumer credit. This provided a major opportunity for the rent-to-own (RTO) business model. In less than five years, the industry expanded from hundreds of stores to 3,000 and became established as a viable alternative for subprime consumers. Today, the industry has grown to over 8,600 stores in the U.S., annually serving over four million customers, and generating over $7.6 billion in revenue (APRO 2011).

RTO is a financial mechanism allowing consumers immediate access to merchandise—most commonly appliances, electronics, or furniture—with neither credit check nor down payment in exchange for making a series of payments. Consumers have the flexibility to choose their payment periodicity, most commonly weekly or monthly. The customers are typically comprised of the "working poor"—a customer base which is generally young and while employed have low income (see, e.g., Anderson and Jaggia (2009) and FTC (2000)). The agreement is for a fixed time period, usually twelve to twenty-four months; at the same time, the customer maintains the ability to terminate the arrangement at any point either by returning the item—making it into a rental transaction—or by making a final lump sum payment. Should all payments be made, or the early purchase option utilized, the customer takes ownership of the merchandise. However, no adverse credit action occurs if the consumer decides to terminate after only one payment or after just a few.[1]

Clearly, following the market crash in 2008, which was predicated in part on fallout from (subprime) mortgage-backed securities, more attention has been focused on subprime borrowing in general. Because RTO customers are predominately lower income and/or financially constrained,[2] RTO is generally viewed as part of the subprime financial industry along with businesses like check-cashing firms, payday lenders, and pawn shops. The initial reaction, when faced with the very high annual percentage rates associated with subprime financing, may well be that such

[1]Another commonly offered option is reinstatement. If for some reason a consumer temporarily suspends the agreement—ceasing payments and returning the item—stores typically allow the later resumption of the contract with credit given for past payments (possibly full credit). Often, the customer can choose to upgrade merchandize.

[2]Both Jappelli (1990) and Gross and Souleles (2002) report that around 20 percent of US residents are credit constrained.

lending must be usurious and exploitative. However, as the current paper illustrates in the case of RTO, the challenges faced by serving such consumers are very different than those faced dealing with unconstrained or prime borrowers.

In this paper, we develop a rational-expectations competitive equilibrium model to explore the pricing mechanism of a RTO agreement. Our model accounts for the unique features of the RTO business model, namely the embedded options of return and early purchase.[3] We depart from standard option frameworks that rely on a no-arbitrage argument to price contingent claims since, unlike financial securities, RTO products are illiquid, indivisible and a customer's decision to exercise an option is often dictated by idiosyncratic (income or taste) shocks. How customers choose to exercise these embedded options is fundamental to the analysis of the financing costs. We find that these choices depend on the length of the contract as well as the frequency of the payments (weekly or monthly). We also include several free services offered in an RTO contract, such as delivery, set-up, maintenance and pick-up (if necessary), which represents significant value to the customer.

In most of the RTO literature, reported financing costs are based on anecdotal evidence where the retail price of a few selected items are compared with the price charged by RTO stores. In this paper, we derive observed markups and APRs from our data set that includes a large number of actual transactions for several product types and contract lengths. These observed markups and APRs provide reliable estimates of the (market) financing costs of RTO agreements. We then compare these observed values with the predicted markups and APRs generated by our model after calibration. First we assume that consumer behavior, as it relates to the exercise of the embedded options, follows a simple iid option exercise strategy. This allows us to derive some useful insights about the associated financing costs of RTO agreements and generate an interesting benchmark that enables a rich analysis of customers' behavior later in the paper. Since the empirical data do not support the simple iid pattern, we extend our model allowing for time dependent option exercise probabilities.

Our data set is drawn from proprietary information from four stores of a small RTO chain in the Southeast. It consists of 7,984 transactional records of monthly and weekly agreements originating from August 23, 2001 to August 6, 2004. We find that the majority of contracts end as a rental with a much greater probability of return under weekly contracts as compared to monthly contracts (73% vs. 52%). Of those who end up acquiring the item, about half of

[3]For an analysis of options embedded in lease contracts under perfect capital markets; see, for instance, McConnell and Schallheim (1983) and Grenadier (1995). Note that, unlike for RTO agreements, the embedded options offered by lease contracts are of the European type, which greatly simplifies the derivation of the results.

them make payments to term with the other half exercising the early purchase option. We also note that the probability of return is relatively high in the initial part of the contract, declining steadily thereafter. This suggests that a substantial portion of customer base values the return option, some using RTO purely as a short term rental mechanism. We can also infer from the above weekly versus monthly finding that there is self selection in that customers who expect to be renters will prefer their agreements to require weekly payments. The estimated early purchase probability spikes around three months after contract origination and another around three months before its conclusion. The earlier spike, for the most part, is dictated by a commonly offered promotion whereby an RTO customer may make a purchase, interest free for 90 days, at a predetermined cash price. Another explanation for the earlier spike is the experimentation effect where the consumer uses RTO to gauge the utility of the item, the length of the need, and the ability to make payments. The latter spike is probably due to the fact that as time passes, the extra cost of exercising the early payment option is relatively small.

We calibrate our model for several products categories, contractual lengths and payment periodicity. For reasonable parameter values, the model is able to generate very large APRs, most notably for short term contracts with weekly payment schedules. We note that the APR is an accurate measure of ex post cost only to the portion of the customer base which pays to term. Our results highlight how free services coupled with a high incidence of return and early purchase, especially in the early stages of the contract, raises the financing costs of a pay to term customer for whom an RTO may be the only feasible way of acquisition. Overall, our model is able to match the patterns displayed by the observed APRs very well; however, the model falls a little short in matching the levels.

While still relatively under-researched, the literature on RTO has expanded in recent years. Hill et al (1998) and Walden (1990) present evidence of the high cost of RTO. Both Anderson and Jackson (2001) and Zikmund-Fisher and Parker (1999) explore the motivation behind users of RTO. Lacko et al (2002) and (2003) employ survey data to examine the characteristics, experiences, and purchase behavior of RTO customers. Beales, Eisenach, and Litan (2012) discuss the welfare implications of regulating the RTO industry. Few attempts have been made to explain the reasons for exercising the embedded options.[4] In a recent paper, Czervonko (2012) develops a partial equilibrium structural model to study how RTO customers respond to income shock. Anderson and Jaggia (2012) estimate the return, early purchase, and default probabilities for two distinct groups of RTO customers with differing (unobserved) motives for entering a contract.

[4] Morse (2011) shows that, even though payday lenders may charge interest rates over 400%, payday lending contributes to improve wealthfare for households facing severe financial distress.

To the best of our knowledge, our paper is among the very first attempts to propose a unified theoretical setting for the RTO business and to explore its implications by calibrating the model using a unique database of RTO transactions. Even though the paper remains silent on welfare issues and the reasons why early purchase or return options are exercised, it manages to capture in a simple way the impact of these unique features on the pricing of an RTO agreement. Interestingly, while most of the existing literature depicts RTO agreements as a means of acquisition for a financially constrained population, our calibrations reveal that pay to term customers, for the most part, generate less that a quarter of the total revenue for the RTO store. Overall, the paper contributes to the public policy debate on subprime lending as well as to the economic literature by deepening our understanding of the explicit sources and magnitudes of the costs of RTO arrangements to consumers.

The paper is organized as follows. Section 2 develops a simple theoretical framework that accounts for the unique features of a RTO agreement taking the customer behavior, with regards to return and early purchase, as given. Section 3 summarizes the RTO transactions from our data set and derives the observed markups and APRs by product type and contract length. Section 4 provides estimates for the key parameters which are then used as inputs to generate markups and APRs. In Section 5, the basic model is extended to allow for endogenous exercise of return and early purchase options by customers and calibrated to match the observed customer behavior from our data set. Section 6 concludes. All proofs are gathered in the Appendix.

2 Theoretical Setting

A customer initiating a contract of length N, agrees to a series of equal payments p_N which are scheduled at time k; $k = 0, 1, 2, \ldots, N$. Notice that the contract is structured as an annuity due and so $N + 1$ payments will be due. In practice, RTO agreements typically run one to three years with the payment periodicity being either weekly or monthly, so, e.g., a two year contract will specify 25 (105) monthly (weekly) payments.

2.1 Product and Contract Characteristics

Let v_0 denote the (average) retail value of a product. In order to develop a formal model, we pay attention to the particular attributes associated with an RTO agreement.

As mentioned above, p_N is the periodic payment charged by a RTO store for a product with initial value v_0 rented under an agreement of length N. Over the life of the contract, at any point, the customer maintains a termination option to return the product with no penalty, an early purchase option allowing her to take immediate ownership, as well as the option to simply pay p_N and continue the agreement another period.

RTO stores are generally required to disclose two prices: (i) the cash price, that is the price at which a customer may buy the item over a 90 day period and (ii) the total RTO price, that is the sum of all payments necessary to purchase the item over time. The cash price is $(1 + \pi)v_0$, where $(1 + \pi) > 1$ is the markup over the retail price and the total price is $(N + 1)p_N$.

Before we discuss the options embedded in the contract, it is important to point out that RTO stores provide additional free services to their customers that are not offered by standard retail stores. These services include delivery, set-up, and pick-up, if required.[5] For convenience, we assume that this induces a cost to the store that is proportional to the (initial) retail value of the product with rate $m_S \in [0, 1]$. In other words, we use $m_S v_0$ to approximate the delivery and set-up costs as well as the pick-up and restocking costs. In addition, there are costs related to maintenance and payment enforcement. RTO stores provide free maintenance over the length of the contract, which may require a service call. A service call may also be necessary when a customer fails to make scheduled payments on time.[6] Let $\lambda = \lambda_M + \lambda_E$, be the probability that in any given period,[7] a service call is required either for maintenance, with probability λ_M, or for payment enforcement reason, with probability λ_E. For simplicity, we assume that the associated cost due to maintenance or payment enforcement is also equal to $m_S v_0$.

Obviously, RTO stores incur other costs—such as rent and utilities payments, labor, and merchandise purchase costs and so on—however, we are assuming that these costs are the same as those incurred by conventional retail stores.[8]

[5] In 2012, the operating expense ratio of Rent a Center and Aaron's, representing two major RTO companies, was 58.9% and 42.8%, respectively. These ratios are broadly speaking twice those for retail competitors such as Wal-Mart Stores and Best Buy, for which the corresponding ratios were 19.4% and 22.7%, respectively. The high operating expense ratio is most likely due to the special services offered by RTO stores.

[6] Anderson and Jaggia (2009) report that two of every five scheduled periodic payments are late.

[7] While there is no need for payment enforcement in the initial period, this period is associated with a higher demand for maintenance. Furthermore, assuming that each period the expected cost due to maintenance and payment enforcement is the same greatly simplifies the calibrations.

[8] RTO stores often run promotions such as "rent today with no payments for a month". In this paper, we ignore such promotions as well as the fees such as late payment fees, inhome collection fees, reinstallement fees, and optional waiver fees. Overall, we expect the effect of promotions and fees on the financing costs to be relatively small.

2.1.1 Early Purchase

RTO stores offer a 90 days same as cash promotion whereby a customer may make a purchase, interest free for 90 days, at a predetermined cash price. Let T denote the number of periods that the same as cash promotion applies; for monthly contracts, $T = 3$. The customer may exercise the early purchase option at any date $k \leq N$—note, at that point, the product will have been used for k periods (with k payments already made)—by making additional lump sum payment (purchasing option price)

$$D_k = \begin{cases} (1+\pi)v_0 - kp_N, \ 1 \leq k \leq T, \\ \varpi_{k,N} p_N, \ T+1 \leq k \leq N, \end{cases} \tag{1}$$

where $\varpi_{k,N} = \max\{1, \varpi(N+1-k)\}$ and $\varpi \in [0,1]$. In practice, we observe that $\varpi \in [0.5, 0.6]$, i.e., one gains ownership by paying about one half of the remaining payments.[9] An early purchase option can be seen as an American call option with a time dependent strike price equal to the early purchase lump sum payment.

2.1.2 Return and Default

In terms of absolute numbers, most RTO contracts conclude without the underlying merchandise being purchased, i.e., the outcome is either a return or a default. The customer holds an option, namely an American put option with a zero strike, to return the merchandise at any date $k \leq N$ at her discretion. We do not differentiate between the reasons why customers choose to exercise this option. Some consumers might have entered into a RTO contract to satisfy their short term rental needs while others might be responding to a change of their financial situation (income shock) or perceived benefits from using the product (taste shock). If exercised, the store reclaims the (used) merchandise and experiences an associated decline in the economic value of that item; there is depreciation. We associate two distinct scenarios with default: either the customer "skipped" (stopped making payments but did not return the item) or, after the item was returned, it was found to be of zero additional economic value to the store.

[9] RTO regulation on early purchase payment differs across states. For instance, in Minnesota, the consumer may acquire the merchandize by paying 55% of the difference between the total scheduled payments and the total amount already paid. In California, the customer may acquire the merchandize by paying an amount equal to the cash price multiplied by the ratio of the number of remaining periodic payments to the total number of scheduled periodic payments, whereas in New York state, she may get ownship by paying the cash price minus fifty percent of all previous periodic payments made. Both California and New York state laws mandate that the total rent should not exceed 2.25 times the cash price. (see Winn 2011).

We assume the merchandise underlying the RTO agreement is subject to (geometric) depreciation at a rate of $\delta \in (0, 1]$ per period. Further δ comprises two economic forces, i.e., $1 - \delta = (1 - \delta_P)(1 - \delta_O)$. The first, δ_P, is physical depreciation due to general wear-and-tear; the second, δ_O, represents real or perceived obsolescence due, e.g., to changing tastes or the entrance of improved alternative products.

Considering those contracts that conclude with no purchase, let $\psi \in [0, 1]$ denote the proportion wherein the customer defaulted and so $1 - \psi$ is the proportion which ended with merchandise return. It follows that should such a conclusion occur at (the beginning of) date k, the net value of the merchandise from the standpoint of the RTO store is $(1 - \psi)v_k$, with $v_k = (1 - \delta)^k v_0$.

2.1.3 Transition Probabilities

The RTO industry serves, for the most part, lower income customers. Some customers may have the luxury of exercising their options optimally; however, for the vast majority of customers, utility derived from the product, financial constraints and, in some cases, expensive services offered along with the product may dictate consumer behavior. Here we discuss the probabilities that these options are exercised.

The transition probabilities are fundamental to the paper's analysis as they capture the customer use of the various options embedded in the RTO agreement. In this section, we assume that the transition probabilities are stationary, which allows us to derive a closed form solution for the periodic payment. Further, the assumption enables us to generate some useful insights on the associated financing costs of RTO and generate an interesting benchmark that enables a rich analysis of customers' behavior later in the paper.

At each date, there are three possible outcomes for the contract: one, the contract continues with the customer making her payment, this occurs with constant probability q_M; two, with constant probability q_H, the contract concludes with the early purchase option exercised; or, three, with constant probability q_L, the contact concludes but there is no purchase. As noted above, the third possibility can occur either via merchandise return or customer default. In our model the probability of return is $(1 - \psi)\, q_L$ while that of default is ψq_L.

2.2 Value of the Contract

Following Rothschild and Stiglitz (1976), we assume that individual RTO stores are risk neutral, in that they are only concerned with expected profits and are all identical.[10] Further, they have a time preference discount factor $\beta \in (0,1)$ equal to the reciprocal of one plus their cost of capital. Consider a contract of length N on a product with periodic payment p_N and initial (retail) value v_0. Let V_{N-k} be the (continuation) value to the RTO store of this contract after $k+1$ payments have been made; note that $V_0 = 0$. By the definition of the continuation value V_N, we have

$$V_N = \beta \left[q_M(V_{N-1} + p_N - \lambda m_S v_0) + q_H D_1 + q_L(1-\psi)v_1 - q_L m_S v_0 \right].$$

More generally, for $1 \le k \le N$,

$$V_{N-k+1} = \beta \left[q_M(V_{N-k} + p_N - \lambda m_S v_0) + q_H D_k + q_L(1-\psi)v_k - q_L m_S v_0 \right],$$

which leads to

$$V_N = \sum_{k=1}^{N} q_M^k \beta^k \left[p_N - \lambda m_S v_0 \right] + \sum_{k=1}^{N} q_M^{k-1} q_L \beta^k \left[(1-\psi)v_k - m_S v_0 \right] + \sum_{k=1}^{N} q_M^{k-1} q_H \beta^k D_k.$$

The continuation value V_N is simply the cumulative value of the expected discounted payoffs, where at each period, the expected payoff depends on the probabilities of exercising (or not exercising) the embedded options offered by the contract. In the Appendix, we show that, for $\varpi \ge 0.5$, separating the terms that contain p_N from those that contain v_0, V_N can be written as:

$$\begin{aligned} V_N = {} & \beta \left[q_M I_N(\beta q_M) + \varpi q_H J_N(\beta q_M) + q_H(1-\varpi)(\beta q_M)^{N-1} \right] p_N \\ & - \beta q_H \left[\varpi(N+1) I_T(\beta q_M) + (1-\varpi)(\beta q_M)^{T-1} J_T(1/(\beta q_M)) \right] p_N \\ & - \beta \lambda q_M I_N(\beta q_M) v_0 \\ & + \beta q_L \left[(1-\psi)(1-\delta) I_N(\beta q_M(1-\delta)) - m_S I_N(\beta q_M) + q_H(1+\pi) I_T(\beta q_M) \right] v_0, \end{aligned}$$

with $I_N(a) = \frac{1-a^N}{1-a}$ and $J_N(a) = \frac{a^{N+1}-(N+1)a+N}{(1-a)^2}$.

Next, in order to determine the periodic payment p_N, we assume a competitive equilibrium in which individual firms can either sell a product at retail price v_0 or offer it as an RTO product.

[10] For simplicity, our model treats RTO contracts in isolation. In reality, RTO stores hold a portfolio of contracts (contingent claims) written on underlying assets, namely the products, with different maturities. This allows them to diversify away part of the idiosyncratic risk of a single contract. Our assumption regarding risk neutrality is particularly justified if (i) contract outcomes are independent and (ii) stores' total revenues, a proxy for stores' total consumption, remain nearly constant overtime.

Competitive forces ensure that new entries will drive profits to zero. The free-entry condition implies that, at time 0, the continuation value of the contract plus the periodic payment minus the delivery and set-up cost minus the expected maintenance cost total must be equal to the retail price, i.e.,

$$V_N + p_N - m_S v_0 - \lambda m_S v_0 = v_0. \tag{2}$$

It readily follows that p_N is given by

$$p_N = \Xi_N \times v_0, \tag{3}$$

where the earnings yield Ξ_N is given by

$$\Xi_N = \frac{1 + S_N + M_N - L_N - C_T}{X_N + Y_N - Z_T - B_T},$$

where

$$\begin{aligned}
S_N &= [1 + \beta q_L I_N(\beta q_M)]\, m_S \\
M_N &= \lambda I_{N+1}(\beta q_M) m_S \\
L_N &= \beta q_L (1-\psi)(1-\delta) I_N(\beta q_M (1-\delta)) \\
X_N &= I_{N+1}(\beta q_M) \\
Y_N &= \beta q_H \left[\varpi J_N(\beta q_M) + (1-\varpi)(\beta q_M)^{N-1}\right] \\
C_T &= \beta q_H (1+\pi) I_T(\beta q_M) \\
B_T &= \beta q_H (\beta q_M)^{T-1} J_T(1/(\beta q_M)) \\
Z_T &= \beta q_H \varpi \left[(N+1) I_T(\beta q_M) - (\beta q_M)^{T-1} J_T(1/(\beta q_M))\right].
\end{aligned}$$

The above values are evaluated at time 0. S_N and M_N represent the free delivery and set-up costs (also pick-up and restocking if necessary), and maintenance and payment enforcement costs, respectively. L_N is the depreciated value of a returned product. X_N is the value of periodic payments over the lifetime of the contract. Finally, the value of the lump sum payment from an early purchase is given by $(C_T - B_T) + (Y_N - Z_T)$, where $C_T - B_T$ and $Y_N - Z_T$ are the values if the early purchase[11] occurs during and after the 90 days same as cash period, respectively.

Clearly, there is not a unique pricing couple (π, p_N) compatible with a zero profit condition. As expected, relationship (3) indicates that the periodic payment p_N is decreasing in π. As the main focus of the paper is to investigate whether observed markups and APRs can be rationalized

[11]Note that Y_N is the value of the lump sum payment from an early purchase in absence of the 90 days same as cash promotion; B_T and C_T capture the impact of the 90 days same as cash promotion.

by the unique (and costly) features of RTO agreements, for most of our analysis, we set π equal to the lowest reasonable value, i.e., $\pi = m_S + M_T$. Here, the markup π over the retail price only covers the delivery and set-up costs as wells as the maintenance and payment enforcement costs incurred during the 90 days same as cash period; a higher value of π only lowers the resulting markups and APRs.

2.2.1 Markup

We define the markup θ_N on a contract of length N as the ratio of the total amount paid if the customer pays to term relative to the product's retail price, i.e.,

$$\theta_N = \frac{(N+1)p_N}{v_0}.$$

Using relationship (3) we have

$$\theta_N = (N+1) \times \Xi_N. \tag{4}$$

In the special case of every customer paying to term ($q_M = 1$) we have:

$$\theta_N = (N+1)\left[\frac{(1+m_S)(1-\beta)}{1-\beta^{N+1}} + \lambda m_S\right],$$

which can be shown to increase with the contractual length N.

2.2.2 Average Percentage Rate (APR)

Let r_N denote the interest rate per period for a contract of length N. By construction, APRs assume payment to term, consequently r_N solves

$$\begin{aligned} v_0 &= \sum_{k=0}^{N} \frac{p_N}{(1+r_N)^k} \\ &= p_N \frac{1+r_N}{r_N}\left[1 - \frac{1}{(1+r_N)^{N+1}}\right]. \end{aligned}$$

The average percentage rate APR_N is, e.g., then twelve times r_N with a monthly payment schedule. Recalling that $\theta_N = \frac{(N+1)p_N}{v_0}$, we note that markup θ_N and interest rate r_N are linked by the following relationship

$$\frac{1+r_N}{r_N}\left[1 - \frac{1}{(1+r_N)^{N+1}}\right] = \frac{N+1}{\theta_N}. \tag{5}$$

2.3 Properties of the Contract

In this subsection, we derive some properties of the contract in order to identify the factors that contribute to high financing costs. We make the following assumption:

Assumption A.1. $m_S \leq (1-\delta)^N$

This condition implies that when a product is returned, the pick up cost to the store is less than the residual value of the product. Observe that for monthly contracts, if $N = 24$ and the annual depreciation $\delta_A = 0.5$, we shall have $m_S \leq 0.25$, which we believe is a reasonable upper bound for m_S. This condition is even less binding for smaller values of N and δ.

Assumption A.2. Either $\frac{T}{N}$ or $\frac{q_H}{q_M}$ is sufficiently small.

This condition implies that either (i) the 90 days same as cash promotion represents a short period of time relatively to the length of the contract or (ii) the frequency of early purchases is not too large. Both requirements are empirically satisfied.

Property 1: p_N is increasing in λ, ψ, δ and m_S and decreasing in ϖ, β and N.

The results are very intuitive. The payment p_N can be viewed as a measure of the cost of providing the features of RTO to this customer base given general business costs and probable contract usage. An increase in λ implies a greater cost due to maintenance and payment enforcement; an increase in ψ means a greater problem with customer default. A higher δ means the merchandise depreciates faster and so subsequent re-rental income (if necessary) would be less. The greater is ϖ the higher is the consumer cost of exercising the early purchase option and so the greater are the proceeds to the store. A lower β is associated with the store having a higher cost of capital and so it would need to charge a higher implicit financing rate. Finally, it is not surprising that the longer the period involved, the smaller the periodic payment.

The markup θ_N inherits all the properties of p_N, except the ones involving the contract length N. As larger length contracts offer more flexibility, one might expect the markup to generally be increasing in the length of the contract.[12] More specifically, one can show that as the contract length N increases, the markup rises (declines) whenever $\varepsilon_{p_N} < \frac{N}{N+1}$ ($\varepsilon_{p_N} > \frac{N}{N+1}$), where $\varepsilon_{p_N} = -\frac{\partial p_N}{\partial N} \times \frac{N}{p_N} > 0$ is the elasticity of the periodic payment with respect to the contract length. Financially constrained customers may be willing to accept a higher markup in order to spread over time the cost of the RTO.

Property 2: The average percentage rate APR_N is increasing and convex in markup θ_N; the

[12] Although we do not report further results, if β, q_M and ϖ are large enough while δ is small, then θ_N and N may be linked by a U-shape relationship.

effect of an increase in the contract length N on the APR is given by the sign of the following quantity $O_N - \varepsilon_{p_N}$ where $O_N = N \ln(1 + r_N)/((1 + r_N)^{N+1} - 1)$.

The convexity of the APR in the markup indicates that, fixing N, a small increase in p_N may significantly raise the financing costs. The term O_N captures the pure effect of an increase in the number of periods (fixing r_N and p_N); its impact on the APR is positive. We conclude that the APR rises (falls) with N whenever ε_{p_N} is sufficiently small (large).

3 Data Analysis

The data set is drawn from proprietary information from four stores of a small RTO chain in the Southeast. It represents all available transactional records as of the date gathered and was filtered only to remove personal and confidential information. It consists of 7,984 records of monthly and weekly agreements originating from August 23, 2001 to August 6, 2004.

3.1 Contract Outcomes and Characteristics

Table 1: Contract Outcome by Merchandise Category (in Percentages)

	All	Appliances	Computers	Electronics	Furniture	Jewelry
Pay to Term	12.39	11.86	9.3	10.48	13.75	17.88
Early Purchase	11.94	11.86	4.94	10.05	15.11	10.35
Return	72.53	74.64	82.27	75.94	68.51	62.59
Default	3.14	1.63	3.49	3.53	2.62	9.18
Observations	5,930	1,534	344	1,642	1,985	425

(b) Monthly Contracts

	All	Appliances	Computers	Electronics	Furniture	Jewelry
Pay to Term	21.03	20.47	19.72	19.52	22.45	22.12
Early Purchase	23.95	23.59	11.27	19.96	28.36	21.15
Return	51.61	54.60	67.61	55.31	45.56	48.08
Default	3.41	1.34	1.41	5.21	3.63	8.65
Observations	2,054	674	71	461	744	104

Table 1 presents, within each merchandise group and overall, the percentage frequency of the various conclusions to the agreements.

Notice about three fourths of the agreements are on a weekly schedule (5,930 versus 2,054). Overall, the most common outcome is return with 72.53% (51.61%) of weekly (monthly) customers having ex post used RTO as a rental agreement. Those on weekly schedules are only about half as likely to end up owning the underlying item (24.33% versus 44.98%). This suggests that there is pre-selection of payment schedules by customers; those expecting to rent prefer a weekly agreement arrangement for the greater flexibility offered while whose expecting to purchase prefer a monthly agreement for the greater convenience (Anderson and Jaggia (2012)). Regardless of payment schedule, when a purchase occurs, about half the time it is by making "payments to term," i.e., by completing the payment schedule, the other half is by exercising the early purchase option. Observe that the incidence of default, and so implicitly the business risk present, is 3.14% of weekly and 3.41% of monthly contracts.

Considering the merchandise specific results, we see some interesting variation. For instance, returns are highest for computers and lowest for furniture and jewelry. Also, there is considerable variability in default rates, being highest for jewelry and lowest for appliances. This is suggestive that consumer behavior is, in part, merchandise-specific informed by, e.g., the utility derived, merchandise portability, and the existence of a secondary market.

Table 2: Contract Outcome by Contract Length (in Percentages)

(a) Weekly Contracts

	6 m	12 m	18 m	24 m
Pay to Term	21.19	9.08	9.76	9.88
Early Purchase	26.57	12.87	6.64	3.09
Return	51.04	73.56	79.83	85.80
Default	1.19	4.48	3.77	1.23
Observations	335	870	1,988	162

(b) Monthly Contracts

	6 m	12 m	18 m	24 m
Pay to Term	46.90	19.42	17.93	12.90
Early Purchase	23.01	30.58	25.35	12.90
Return	28.32	47.84	52.66	72.58
Default	1.77	2.16	4.06	1.61
Observations	113	278	714	62

Table 2 presents, for each contract length, the percentage frequency of the various conclusions to the agreements.

Return probabilities are higher for longer agreements and for weekly payment schedules. For example, the return probability on weekly (monthly) schedules steadily increases from 51.04%

(28.32%) to 85.80% (72.58%) for 6-month and 24-month contracts, respectively. For the most part, pay to term as well as early purchase customers prefer shorter agreements and monthly schedules. Finally, early purchases are generally associated with shorter agreements and weekly schedules. These results are consistent with the earlier explanation that the choice of the contract terms depends on whether the consumer ex ante expects to be a renter or a buyer.

Table 3: Contract Length Categorized by Outcome

	Weekly			Monthly		
	Maximum	Actual	Obs.	Maximum	Actual	Obs.
Pay to Term	12.22	12.22	735	13.58	13.58	432
	(13.03)	(13.03)		(14.78)	(14.78)	
Early Purchase	10.71	4.08	708	15.19	7.65	492
	(11.13)	(2.12)		(16.03)	(6.17)	
Return and Default	15.89	2.39	4,487	16.79	4.08	1,130
	(17.20)	(1.37)		(17.27)	(2.80)	
All	14.82	3.81		15.73	6.93	
	(16.20)	(1.63)	5,930	(16.50)	(4.423)	2,054

Table 3 presents the mean (median) length in months, both for the maximum contractual length and the actual running length of the agreement.

As expected, actual length is significantly lower than the maximum contractual length—except, obviously, for those that are paid to term. The average maximum length for all contracts is 14.82 months for weekly contracts and 15.73 months for monthly contracts. We see, regardless of outcome, contracts on a weekly schedule resolve quicker than those that pay monthly. It is noteworthy that returns and defaults happen quite early in the contract. Further, early purchases occur in a little under twice the time taken by returns and defaults. From Table 1 we know that early purchase is less likely under a weekly schedule but here we see that when it does happen, it occurs relatively earlier in the contractual period than is the case with monthly agreements. Also very interesting is the differences in contract length across these subgroups. In particular, the maximum length associated with return and default is relatively short.

3.2 Observed Markups and APRs

Clearly, RTO is an expensive service to offer. For example, Swagler and Wheeler (1989) report monthly APRs up to 150% for monthly contracts and APRs up to 190% for weekly contracts while Zikmund-Fisher and Parker (1999) report APRs of 230%. Other popular sources, including Consumer Reports, (Consumerreport.org, June 25, 2011) inform that RTO stores often charge two to three times the amount it would cost to buy an item from a traditional retail store.

In this paper, we are also able to estimate the average markups and APRs directly from our data. For most transactions, we have the information on the purchase cost to the store for each item. Given the presence of outliers in our data, we focus only on the items for which the ratio of the maximum payment amount and the purchase cost is less than five. We then use this purchase cost to approximate the relevant retail price, by using the estimated annual gross margin as a percentage of sales in the U.S. (http://www2.census.gov/retail/releases/current/arts/gmper.xls). These gross margins do not include expenses such as rent or salaries and are, therefore, appropriate to approximate the retail price per item. A proxy $\widehat{v}_0$ for the retail price is simply given by

$$\widehat{v}_0 = \frac{c}{1 - m_g},$$

where c is the purchase cost and m_g is the gross margin for the item. We use $\widehat{v}_0$ along with the contract length N and the periodic payment p_N to calculate markups and APRs. The average gross margins, markups, and observed APRs are presented in the following table.

Table 4: Markup and APR per Merchandise Category and Contract Length

(a) Merchandise Category

	All	Appliances	Electronics	Furniture
Gross Margin	34.87%	27.60%	27.60%	46.50%
APR	213%	232%	235%	180%
Markup	2.64	2.82	2.78	2.37
Periodic Payment	0.165	0.176	0.174	0.148
Observations	5,335	1,763	1,577	1,995

(b) Contract Length

	6 m	12 m	18 m	24 m
Gross Margin	34.87%	34.87%	34.87%	34.87%
APR	275%	239%	206%	160%
Markup	1.66	2.36	2.90	3.05
Periodic Payment	0.237	0.182	0.153	0.122
Observations	160	872	2,056	112

Table 4 presents, the observed average APRs, markups and periodic payments (in percentages of the estimated retail value) within each merchandise category and contract length.

Overall, we find that the average APR for all items is slightly over 200%. The average values are slightly lower for furniture as compared to appliances and electronics. It is noteworthy that the average markups and APRs generated from our data set are generally consistent with those reported in the extant literature (see, e.g., Zikmund-Fisher and Parker (1999)).

While markups are found to be rising with the contract length N, in contrast, the average APRs and periodic payments are monotonically declining in N. Arguably, RTO stores focus on periodic payments rather than APRs as the former are more visible to customers. Consequently, stores restrain from raising too much the markup value as the contract length increases. For instance, in order to maintain the 6 month APR of 275%, the markup required for an 18 month contract would be 3.57, which is well above the 2.90 value reported in Table 4.

As discussed in Property 1, a rising markup with the contractual length indicates that the elasticity ε_{p_N} is less than one. At the same time, a decreasing APR with the contractual length reveals that ε_{p_N} must remain above a floor level O_N (Property 2). Given the reported values of the APR in Table 4, we infer that O_N ranges from 0.14 to 0.38.

4 Calibrations for the Basic Model

We infer the transition probabilities from the estimated cumulative probabilities, shown in Tables 1 and 2. Let (Q_H, Q_M, Q_L) denote the cumulative probabilities of an early purchase, payment to term and return or default, respectively. Given our definition of an early purchase lump sum payment, a RTO customer is indifferent between exercising the early purchase option at date N and making the final periodic payment. This implies that $Q_M = q_M^N$ and at date $k \leq N$, the probability of an early purchase (return) is $q_M^{k-1} q_H$ ($q_M^{k-1} q_L$). As events are independent, we have

$$Q_H = \sum_{k=1}^{N} q_M^{k-1} q_H = q_H \frac{1 - q_M^N}{1 - q_M} \tag{6}$$

$$Q_L = \sum_{k=1}^{N} q_M^{k-1} q_L = q_L \frac{1 - q_M^N}{1 - q_M}. \tag{7}$$

Solving relationships (6) and (7) for the transition probabilities, we get

$$\begin{aligned} q_M &= Q_M^{1/N}, \\ q_H &= Q_H \frac{1 - Q_M^{1/N}}{1 - Q_M}, \\ q_L &= Q_L \frac{1 - Q_M^{1/N}}{1 - Q_M}. \end{aligned}$$

4.1 Early Purchase Payment Lump Sum Amount

Exercising the early purchase option—thereby obtaining immediate ownership—requires one to pay a proportion ϖ of the remaining payments. The parameter ϖ is an important input to our theoretical model. We estimate this parameter by running a regression of the amount actually paid at the time of the option exercise on the total (undiscounted) remaining rent payments.

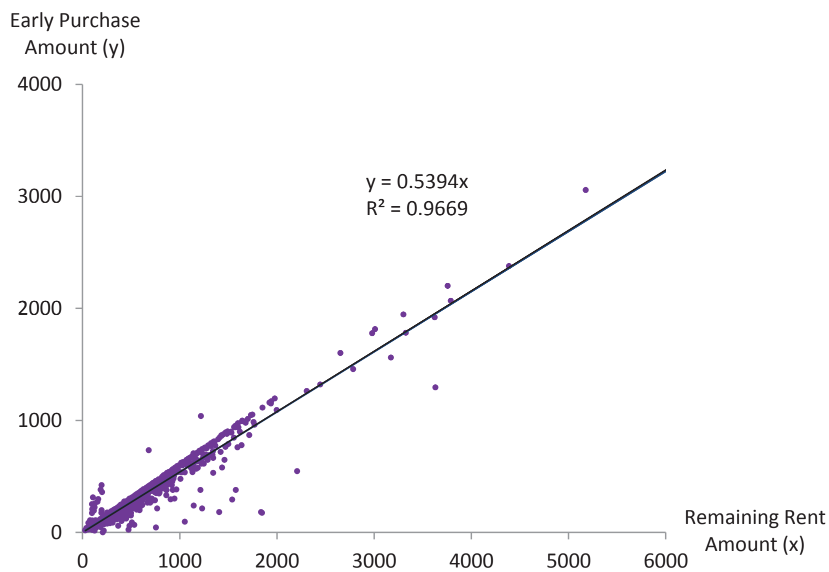

Figure 3 shows the regression of the early purchase option amount on the total remaining rent at the time of option exercise (in dollars).

As illustrated in Figure 3, with an R^2 of 0.9669, the preferred model has a zero intercept (as desired) and yields a slope coefficient (ϖ estimate) of 0.5394. We henceforth take ϖ to be 0.54 and further note that this estimate is consistent with the extant literature, see, e.g., Anderson and Jaggia (2012) or FTC (2000).

4.2 Depreciation

The annual depreciation rate δ_A measures the decline in secondary market value relative to the initial cost. As discussed earlier, this is driven, in part, by the item's physical depreciation due to wear-and-tear and real or perceived obsolescence due to changing tastes or the entrance of improved alternative products. In terms of physical depreciation, one might think computers and electronics would depreciate faster than appliances which, in turn, are faster than furniture and jewelry. By contrast, obsolescence considerations might suggest that the fastest to slowest grouping is computers and electronics, furniture, appliances and then jewelry. Overall, we expect depreciation δ_A to be the largest for computers and electronics and smallest for jewelry.

For the analysis, we use $\delta_A = 0.2$, 0.3 and 0.4 representing low, medium and high rates of depreciation respectively. Our choice of these depreciation rates is guided, in part, by reference to the rates reported by the Bureau of Economic Analysis (BEA) for durable goods owned by consumers; for computers, which is not reported by the BEA, we refer to Doms, Dunn, Oliner and Sichel (2004). For each product, the BEA computes the annual depreciation rate as the ratio of the declining balance rate, which is generally 165%, and the service life of the product in years.[13] Since RTO stores are likely to face significantly higher depreciations of their products than regular households, we adjust the BEA rates by altering the service life of the good. Table 5 displays the depreciation rates reported by the BEA as well as the ones we suggest for RTO.

Table 5: Depreciation Rates and Service Life by Merchandise Category

(a) Households (BEA)

	Appliances	Computers	Electronics	Furniture	Jewelry
Depreciation Rate (%)	15	34	18.33	11.79	15
Service Life (years)	11	5	9	14	11
Declining Balance Rate (%)	165	NA	165	165	165

(b) RTO Stores (Suggested)

	Appliances	Computers	Electronics	Furniture	Jewelry
Depreciation Rate (%)	33.33	55	41.25	20.62	18.33
Service Life (years)	5	3	4	8	9
Declining Balance Rate	165	NA	165	165	165

Table 5 shows the values of the depreciation rate and service life per merchandise category as reported by the BEA and our suggested estimates for RTO stores.

4.3 Service Costs

In our model, the parameter m_S captures service costs to the store. Unlike retail store, RTO customers are offered free delivery, set-up, maintenance and free pick-up, if required. The costs

[13]Whenever possible, the BEA uses resale market data to evaluate price declines of used products. The declining balance method provides a relatively high depreciation charge in the first year of the product's life, which gradually declines over the subsequent years.

depend, e.g., on the average delivery radius and the prevailing wage for employees. This cost also depends on the physical bulk of an item, making delivery costs higher for both appliances and furniture. Other costs relate to the likelihood that the item will be returned (and so the store will have to pick it up). While computers and electronics have a lower delivery cost, they are also associated with a higher likelihood of return. Considering a spectrum of possibilities, at one end is jewelry where no delivery, or set-up would be expected; at the other end, would be major appliances. For the analysis, we use $m_S = 0.10$, 0.15 and 0.20 representing low, medium and high service costs, respectively. As mentioned earlier, we use $m_S v_0$ to approximate the delivery and set-up costs as well as the pick up and restocking costs. In addition, RTO stores incur costs when a service call is required either for maintenance or for payment enforcement. Recall that λ is the probability that in any given period, a service call is required; for simplicity, we assume that the induced cost is also $m_S v_0$. For the calibrations, we set $\lambda = 0.1$. This estimate is based on the analysis of product warranties, informal discussion with RTO store managers and the analysis of late payment behavior documented in Anderson and Jaggia (2009).

4.4 Predicted APRs, Markups and Periodic Payments

Tables 6 and 7 provide the predicted APRs, markups and periodic payments associated with a range of values of δ_A and m_S. We also assume a cost of capital of 15%, so that the (monthly) discount factor β is equal to 0.9884. In addition, the cash price markup over the retail price is set to the lowest reasonable value, i.e., $\pi = m_S + M_T$. Finally, since our focus includes comparing consumer behavior related to weekly versus monthly payment schedules, we use monthly probability estimates for both payment schedules to make the comparison more seamless.

In both tables, the results are grouped by payment schedule (weekly and monthly). In Table 6, the results are provided overall and for appliances, electronics and furniture (the other two categories, with relatively fewer agreements, were omitted for space considerations). As the overall contract length is roughly fifteen months, we set $N = 15$ for the analysis. In Table 7, the results are provided for contract lengths of six, twelve, eighteen and twenty-four months.

Table 6 shows that the APRs and markups increase with the depreciation rate δ_A and the delivery and service parameter m_S. For example, for weekly contracts with $m_S = 0.10$, the APR for all products increases from 52.61% up to 75.33% as δ_A increases from 0.20 to 0.40. At the same two values for δ_A, the corresponding APRs for $m_S = 0.20$ are 125.38% to 146.28%, respectively.

From Table 5, we infer that the appropriate annual depreciation rates for appliances, electronics, and furniture are about 30%, 40%, and 20% respectively. At these depreciation rates,

Table 6: Model Calibration for APRs and Markups — Weekly and Monthly Schedules

(a) Weekly Contracts

δ_A	$m_S = 0.10$ All	Appl	Elec	Furn	$m_S = 0.15$ All	Appl	Elec	Furn	$m_S = 0.20$ All	Appl	Elec	Furn
0.2	52.61	48.63	54.31	52.17	89.87	86.57	93.71	87.49	125.38	122.63	131.24	121.19
	1.35	1.32	1.37	1.35	1.63	1.60	1.66	1.61	1.90	1.88	1.95	1.87
	0.085	0.083	0.085	0.084	0.102	0.100	0.104	0.101	0.119	0.117	0.122	0.117
0.3	63.79	60.30	66.33	62.49	100.44	97.56	105.05	97.26	135.59	133.23	142.22	130.63
	1.43	1.41	1.45	1.42	1.71	1.69	1.74	1.68	1.98	1.96	2.03	1.94
	0.090	0.088	0.091	0.089	0.107	0.105	0.109	0.105	0.124	0.123	0.127	0.121
0.4	75.33	72.33	78.75	73.15	111.43	108.99	116.88	107.42	146.28	144.32	153.73	140.50
	1.52	1.50	1.54	1.50	1.79	1.77	1.83	1.76	2.07	2.05	2.13	2.02
	0.095	0.094	0.096	0.094	0.112	0.111	0.115	0.110	0.129	0.128	0.133	0.126

(b) Monthly Contracts

δ_A	$m_S = 0.10$ All	Appl	Elec	Furn	$m_S = 0.15$ All	Appl	Elec	Furn	$m_S = 0.20$ All	Appl	Elec	Furn
0.2	56.71	51.83	59.04	59.35	85.67	81.33	89.52	86.60	113.50	109.62	118.82	112.88
	1.38	1.35	1.40	1.40	1.60	1.56	1.62	1.60	1.81	1.78	1.85	1.80
	0.086	0.084	0.087	0.088	0.100	0.098	0.102	0.100	0.113	0.111	0.116	0.113
0.3	63.73	59.37	66.71	65.38	92.38	88.51	96.85	92.39	120.01	116.57	125.93	118.51
	1.43	1.40	1.45	1.45	1.65	1.62	1.68	1.65	1.86	1.83	1.91	1.85
	0.090	0.088	0.091	0.090	0.103	0.101	0.105	0.103	0.116	0.115	0.119	0.115
0.4	70.99	67.15	74.64	71.63	99.34	95.96	104.45	98.40	126.79	123.80	133.33	124.36
	1.49	1.46	1.51	1.49	1.70	1.67	1.74	1.69	1.91	1.89	1.96	1.89
	0.093	0.091	0.095	0.093	0.106	0.105	0.109	0.106	0.120	0.118	0.123	0.118

Table 6 presents, for contracts on a weekly and monthly payment schedules, the APR and, directly below, first the markup, and then the periodic payment (in percentages of the retail value), associated with various values of δ_A and m_S. The information is presented for all contracts (All) and for Appliances, Electronics and Furniture. The calibration assumes $\varpi = 0.54$, $\lambda = 0.10$, a cost of capital of 15% and uses constant transition intensities.

Table 7: Model Calibration for APRs and Markups, Weekly and Monthly Payment Schedules

(a) Weekly Contracts	$m_S = 0.1$				$m_S = 0.15$				$m_S = 0.2$			
δ_A	6 m	12 m	18 m	24 m	6 m	12 m	18 m	24 m	6 m	12 m	18 m	24 m
0.2	29.04	58.92	55.44	50.64	95.43	107.81	90.34	78.81	161.02	154.76	123.49	105.44
	1.07	1.31	1.46	1.57	1.24	1.59	1.78	1.94	1.41	1.87	2.10	2.30
	0.153	0.101	0.077	0.063	0.178	0.122	0.094	0.077	0.202	0.144	0.110	0.092
0.3	40.36	72.04	67.13	61.58	106.54	120.29	101.34	89.06	172.09	166.92	134.12	115.30
	1.10	1.39	1.56	1.71	1.27	1.67	1.88	2.07	1.44	1.95	2.21	2.44
	0.157	0.107	0.082	0.068	0.182	0.128	0.099	0.083	0.206	0.150	0.116	0.097
0.4	52.82	85.89	79.01	72.31	118.81	133.56	112.63	99.23	184.34	179.93	145.09	125.16
	1.13	1.47	1.67	1.85	1.30	1.74	1.99	2.21	1.47	2.02	2.31	2.58
	0.162	0.113	0.088	0.074	0.186	0.134	0.105	0.089	0.211	0.156	0.122	0.103

(b) Monthly Contracts	$m_S = 0.1$				$m_S = 0.15$				$m_S = 0.2$			
δ_A	6 m	12 m	18 m	24 m	6 m	12 m	18 m	24 m	6 m	12 m	18 m	24 m
0.2	60.15	58.30	58.30	51.37	106.49	92.28	83.75	75.65	152.53	125.13	108.17	98.71
	1.15	1.31	1.48	1.58	1.27	1.50	1.72	1.89	1.39	1.69	1.95	2.21
	0.165	0.101	0.078	0.063	0.182	0.115	0.090	0.076	0.199	0.130	0.103	0.088
0.3	65.16	65.67	64.88	59.87	111.45	99.37	90.03	83.66	157.48	132.04	114.26	106.43
	1.17	1.35	1.54	1.69	1.29	1.54	1.78	2.00	1.41	1.74	2.01	2.31
	0.166	0.104	0.081	0.068	0.184	0.119	0.093	0.080	0.201	0.133	0.106	0.093
0.4	70.66	73.46	71.58	68.21	116.91	106.88	96.44	91.59	162.92	139.37	120.49	114.11
	1.18	1.39	1.60	1.80	1.30	1.59	1.84	2.11	1.42	1.78	2.07	2.42
	0.169	0.107	0.084	0.072	0.186	0.122	0.097	0.084	0.203	0.137	0.109	0.097

Table 7 presents, for contracts on a weekly and monthly payment schedules, the APR and, directly below, first the markup and then the periodic payment (in percentages of the retail value), associated with various values of annual depreciation δ_A and m_S. The information is presented for all contracts lengths of 6, 12, 18 and 24 months. The calibration assumes $\varpi = 0.54$, $\lambda = 0.10$, a cost of capital of 15% and uses constant transition intensities.

contracts on a weekly payment schedule with $m_S = 0.15$ have the APRs (markups) of about 97.56% (1.69) for appliances, 116.88% (1.83) for electronics, and 87.49% (1.61) for furniture. The corresponding contracts on a monthly payment schedule have slightly lower values for the APRs, markups and periodic payments, but the difference is not very striking. In sum, the APRs and markups are highest for electronics and lowest for furniture, which is consistent with observed APRs and markups, even though the observed values are significantly higher in magnitude.

The predicted APRs, markups and resulting periodic payments, reported in Table 7, are related to contract lengths and payment schedules. As shown in Table 2, the choice of the contracts length depends crucially on whether the consumer ex ante expects to be a renter or a buyer. While the return customers prefer longer term contracts on a weekly payment schedule, both pay to term and early purchase customers prefer shorter contracts on a monthly payment schedule. At the same time, the impact of the delivery and set-up costs as well as the 90 days same as cash promotion is all the more significant when the length of the contract is short.

In general, the APRs for 6- and 12-month contracts are higher than those for 18- and 24-month contracts. Again, as expected, the APRs, markups and periodic payments increase with δ_A and m_S; however, the impact of m_S is more significant, especially for shorter term contracts. For example, given $\delta_A = 0.30$, the APR for a 6-month contract on a weekly schedule, increases from 40.36% to 172.09% as m_S increases from 0.10 to 0.20. A corresponding increases for a 24-month contract is from 61.58% to 115.30%. The difference is probably due to the strong impact of the delivery and set-up costs as well as the 90 days same as cash promotion for shorter contracts. It is also noteworthy that the periodic payment is always decreasing at a decreasing rate with the length of the contract. These findings are consistent with our theoretical results in Property 1.

5 Extension to the Basic Model

So far, for simplicity, our analysis has assumed that each period the exit is determined by a constant, iid triplet of probabilities, (q_L, q_M, q_H). However, our empirical data do not support constant transition probabilities. We find that the transition probabilities vary over the life of the contract, reflecting the realized pattern of return and early purchase option exercises. Consequently, this section extends the model by developing a rational-expectation competitive model that endogenizes option exercises.

5.1 Value of the Contract (revisited)

For $k = 1, 2, ..., N$, let $(q_{L,k}, q_{M,k}, q_{H,k})$ denote the triplet of transition probabilities that in period k the customer terminates the contract with no purchase, makes the contractual payment (and so the contract remains on-going), or makes an early purchase, respectively. Note that the customer must make a payment when the contract is initiated, so $q_{M,0} = 1$ and $q_{L,0} = q_{H,0} = 0$. Let $\overline{q}_{L,k} = \left(\prod_{i=0}^{k-1} q_{M,i}\right) q_{L,k}$, $\overline{q}_{H,k} = \left(\prod_{i=0}^{k-1} q_{M,i}\right) q_{H,k}$ and $\overline{q}_{M,k} = \left(\prod_{i=0}^{k-1} q_{M,i}\right) q_{M,k}$ denote the (unconditional) probabilities that the contract is terminated at period k by no purchase, an early purchase or remains on going, respectively. By the definition of the continuation value V_N, we have

$$\begin{aligned} V_N &= \beta\left[q_{M,1}(V_{N-1} + p_N - \lambda m_S v_0) + q_{H,1} D_1 + q_{L,1}((1-\psi)v_1 - m_S v_0)\right] \\ V_{N-1} &= \beta\left[q_{M,2}(V_{N-2} + p_N - \lambda m_S v_0) + q_{H,2} D_2 + q_{L,2}((1-\psi)v_2 - m_S v_0)\right], \end{aligned}$$

and more generally, by iteration, we obtain

$$V_N = \sum_{k=1}^{N} \overline{q}_{M,k} \beta^k (p_N - \lambda m_S v_0) + \sum_{k=1}^{N} \overline{q}_{L,k} \beta^k \left[(1-\psi)v_k - m_S v_0\right] + \sum_{k=1}^{N} \overline{q}_{H,k} \beta^k D_k.$$

Then, using (1) and (2) leads to the following expression for the earning yield:

$$\Xi_N = \frac{1 + S_N + M_N - L_N - C_T}{X_N + Y_N - Z_T - B_T},$$

where the various components of Ξ_N are now given by: $S_N = \left(1 + \sum_{k=1}^{N} \beta^k \overline{q}_{L,k}\right) m_S$, $M_N = \lambda \left(1 + \sum_{k=1}^{N} \beta^k \overline{q}_{M,k}\right) m_S$, $L_N = (1-\psi) \sum_{k=1}^{N} \left[(\beta(1-\delta))^k \overline{q}_{L,k}\right]$, $X_N = 1 + \sum_{k=1}^{N} \beta^k \overline{q}_{M,k}$, $Y_N = \sum_{k=1}^{N} \left[\beta^k \overline{q}_{H,k} \varpi_{k,N}\right]$, $C_T = (1+\pi) \sum_{k=1}^{T} \beta^k \overline{q}_{H,k}$, $B_T = \sum_{k=1}^{T} \left[\beta^k \overline{q}_{H,k} k\right]$ and $Z_T = \sum_{k=1}^{T} \left[\beta^k \overline{q}_{H,k} \varpi_{k,N}\right]$.

The markup and the APR are derived similarly.

For the analysis, we first employ our data set to calculate the transition probabilities over time. We then feed those estimates into the above model extension for calibrations. For estimation purposes, we partitioned the data into subsets by payment schedule—weekly or monthly—and further by contractual length—six, twelve, eighteen and twenty-four months. Like before, all probabilities are estimated on a monthly basis.

5.2 Observed Contract Outcome Frequency

Table 8: Frequency Distribution — Weekly (in Percentages)

Months	6 Months		12 Months		18 months		24 months	
	Return & Default	Early Purchase	Return & Default	Early Purchase	Return & Default	Early Purchase	Return & Default	Early Purchase
1-3	45.97	22.09	60.80	8.39	62.17	2.26	57.41	1.85
3-6	6.27	4.48	11.15	1.03	12.93	0.40	13.58	0.00
7-9			4.14	1.95	4.38	0.05	10.49	0.00
10-12			1.95	1.49	2.01	0.40	3.70	0.00
13-15					1.21	1.41	1.85	0.00
16-18					0.91	2.11	0.00	0.62
19-21							0.00	0.62
22-24							0.00	0.00
Total	52.24	26.57	78.05	12.87	83.60	6.64	87.04	3.09

Table 8 displays the frequency distribution of returns and defaults and early purchases for several contractual lengths under weekly payment schedules.

Table 9: Frequency Distribution — Monthly (in Percentages)

Months	6 Months		12 Months		18 Months		24 Months	
	Return & Default	Early Purchase	Return & Default	Early Purchase	Return & Default	Early Purchase	Return & Default	Early Purchase
1-3	24.78	12.39	28.42	11.87	27.87	8.26	35.48	3.23
3-6	5.31	10.62	14.75	3.96	14.71	2.24	24.19	0.00
7-9			5.04	7.91	7.56	1.26	1.61	0.00
10-12			1.80	6.83	3.22	3.08	1.61	0.00
13-15					1.40	4.48	6.45	0.00
16-18					1.96	6.02	3.23	0.00
19-21							0.00	6.45
22-24							1.61	3.23
Total	30.09	23.01	50.00	30.58	56.72	25.35	74.19	12.90

Table 9 displays the frequency distribution of returns and defaults and early purchases for several contractual lengths under monthly payment schedules.

Before reporting the transition (conditional) probabilities, we discuss the frequency distribution (unconditional probabilities) shown in Tables 8 and 9. First, as shown earlier in Table 2, regardless of the payment schedule, longer agreements have a higher probability of return and default while shorter agreements have a higher probability of early purchase. Further, for the most part, merchandise under a monthly schedule has a greater chance of early purchase than it does under a weekly schedule. Return and default probabilities, on the other hand, are significantly higher under a weekly schedule. These results are consistent with the earlier explanation that the choice of contract terms depends on whether the consumer ex ante expect to be a renter or a buyer. We also note that the probability of return and default is relatively high in the first few months, declining steadily thereafter. For a 12-month contract on a weekly schedule, this probability is 60.80% during the first 3 months, declining to 11.15%, 4.14%, and 1.95%, respectively, over the subsequent three month periods. The difference between the weekly and monthly payment schedules is especially pronounced during the first three months, where the return and default probabilities are almost twice as high under a weekly schedule. Early purchase probabilities are relatively high in the first three months after contract origination and again close to its conclusion. The earlier spike is probably due to the 90-days same as cash promotion. Another explanation is the experimentation effect where the customer uses RTO to gauge the utility of the item, the length of the need, and the ability to make payments. The latter spike is probably due to the fact that as time passes, the extra cost of exercising the early payment option is relatively small.

As already mentioned, we incorporate the empirical estimates of the time dependent transition probabilities in our theoretical model to produce calibrations on APRs and markups. These probabilities are estimated as

$$q_{H,k} = \frac{N_{H,k}}{N_{k-1}} \text{ and } q_{L,k} = \frac{N_{L,k}}{N_{k-1}},$$

where $N_{H,k}$ and $N_{L,k}$ respectively represent the number of early purchases and returns/defaults in the period k and N_{k-1} represents the number of contracts that have not been terminated as of period $k-1$. Additionally, the parameter ψ is estimated as the number of contracts that defaulted normalized by the number that were "no purchase" (i.e., returns or defaults).[14]

[14] Tables fully detailing the results of this analysis are available upon request from the authors. Note that the analysis was only performed at the aggregate level since disaggregating to the various merchandize groups would have lead to the data being too sparse at some points for reliable estimation.

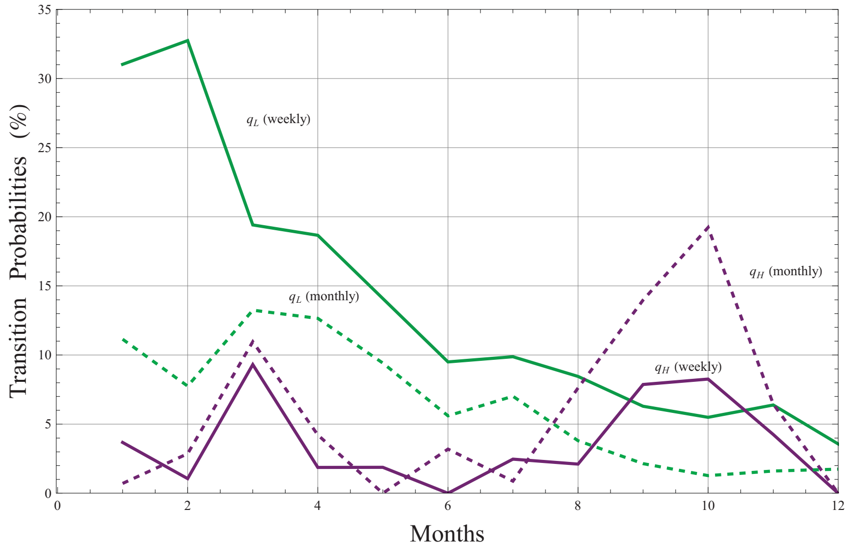

Figure 5: Estimated merchandise early purchase and return probabilities for 12 month contracts.

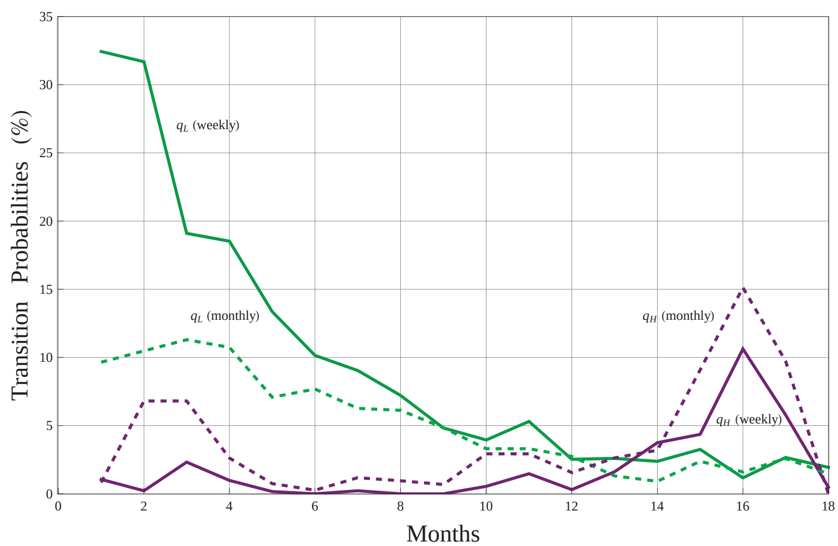

Figure 6: Estimated merchandise early purchase and return probabilities for 18 month contracts.

Figures 5 and 6 illustrate the time dependency of the transition probabilities for RTO agreements under both weekly and monthly payment schedules and for whom the contractual length is either twelve or eighteen months. One reason those lengths were chosen was to match up with the analysis in the previous section which used a fifteen month interval, roughly corresponding to the overall average length. Clearly, as depicted in the figures, these probabilities are not constant and indeed reveal the use of RTO agreement options by customers.

5.3 Predicted APRs, Markups and Periodic Payments

As aforementioned, the transition probabilities account for the optimal or constrained customer behavior; we now calibrate our model to generate APRs and markups and explore the implications. The results are reported in Table 10 for contracts with maximum lengths of six, twelve, eighteen and twenty-four months. The table is obtained using the same assumed depreciation rates and service costs as in Tables 6 and 7; however, we believe that the most reasonable parameter values are $\delta_A = 0.30$ and $m_S = 0.15$. Embedded in each calibration is a $3 \times N$-matrix representing our estimate of $(q_{L,k}, q_{M,k}, q_{H,k})$, with $k = 1, ..., N$ and $N = 6$, 12, 18 and 24

As reported in Table 10, the range of the predicted APRs for weekly contracts is from 80.68% to 254.40%, while for monthly contracts it ranges from 63.34% to 188.28%. The highest APRs are associated with the shortest lengths and highest assumed depreciation and service and delivery costs. Observe that markups and APRs are significantly larger than those reported in Table 7 under the assumption of constant transition probabilities, most noticeably for shorter term contracts. For instance, at our preferred parameter values of $\delta_A = 0.3$ and $m_S = 0.15$, for 12 month contracts under weekly payment schedule, the APR, markup and periodic payment are 182.06%, 2.04 and 0.157 respectively. The corresponding values reported in Table 7 are 120.69%, 1.67, and 0.128, respectively.

Ceteris paribus, APRs and markups are higher for contracts on a weekly payment schedule as opposed to a monthly schedule. Unlike in the case of constant transition probabilities (see Table 7), the difference here is a lot more striking. Since weekly contracts are associated with a higher rate of return than the corresponding monthly contracts (see tables 8 and 9), weekly contracts are expected to be more costly for RTO stores, thus resulting in a higher APR for pay to term customers. Also, since items are generally returned in the early life of the contract, the impact of the depreciation parameter δ_A is somewhat limited. On the other hand, the impact of the free services parameter m_S is large and all the more significant as it governs various costs rather than just the cost related to return.

It is also worth noticing that as the contractual length goes up, in general, markups are rising while APRs and periodic payments are declining, both at a decreasing rate. These patterns are consistent with the ones reported for the observed markups and APRs in Table 4. For the most

Table 10: Model Calibration for APRs and Markups, Weekly and Monthly Payment Schedules

(a) Weekly Contracts	$m_S = 0.1$				$m_S = 0.15$				$m_S = 0.2$			
δ_A	6 m	12 m	18 m	24 m	6 m	12 m	18 m	24 m	6 m	12 m	18 m	24 m
0.2	80.68	109.83	101.02	88.04	157.45	171.17	146.49	127.26	234.28	231.52	190.94	165.44
	1.21	1.60	1.88	2.06	1.40	1.97	2.33	2.61	1.60	2.34	2.78	3.15
	0.172	0.123	0.099	0.082	0.201	0.152	0.123	0.104	0.229	0.180	0.146	0.126
0.3	90.05	121.04	109.83	97.26	166.76	182.06	155.00	136.11	243.68	242.40	199.39	174.23
	1.23	1.67	1.97	2.19	1.43	2.04	2.41	2.73	1.63	2.40	2.86	3.28
	0.176	0.128	0.103	0.087	0.204	0.157	0.127	0.109	0.233	0.185	0.151	0.131
0.4	100.53	133.26	119.32	107.08	177.19	194.01	164.21	145.63	254.24	254.40	208.59	183.74
	1.26	1.74	2.06	2.32	1.46	2.11	2.51	2.87	1.65	2.48	2.95	3.42
	0.180	0.134	0.108	0.093	0.208	0.162	0.132	0.115	0.236	0.191	0.155	0.137

(b) Monthly Contracts	$m_S = 0.1$				$m_S = 0.15$				$m_S = 0.2$			
δ_A	6 m	12 m	18 m	24 m	6 m	12 m	18 m	24 m	6 m	12 m	18 m	24 m
0.2	84.94	73.45	69.49	63.34	132.66	108.92	96.73	90.77	180.24	143.37	122.99	117.01
	1.22	1.39	1.58	1.73	1.34	1.60	1.84	2.10	1.46	1.80	2.09	2.46
	0.174	0.107	0.083	0.069	0.191	0.123	0.097	0.084	0.209	0.139	0.110	0.098
0.3	88.76	79.31	74.71	70.26	136.46	114.58	101.73	97.33	184.05	148.92	127.85	123.37
	1.23	1.43	1.63	1.82	1.35	1.63	1.89	2.19	1.47	1.84	2.14	2.55
	0.175	0.110	0.086	0.073	0.193	0.126	0.099	0.087	0.211	0.141	0.113	0.102
0.4	93.01	85.66	80.22	77.35	140.69	120.73	107.02	104.09	188.28	154.95	133.01	129.96
	1.24	1.46	1.68	1.92	1.36	1.67	1.94	2.28	1.48	1.87	2.19	2.65
	0.177	0.113	0.089	0.077	0.194	0.128	0.102	0.091	0.212	0.144	0.115	0.106

Table 10 presents, for contracts on a weekly and monthly payment schedules, the APR and, directly below, first the markup and then the periodic payment (in percentages of the retail value), associated with various values of annual depreciation δ_A and m_S. The information is presented for all contracts lengths of 6, 12, 18 and 24 months. The calibration assumes $\varpi = 0.54$, $\lambda = 0.10$, a cost of capital of 15% and uses the estimated transition intensities.

part, short term contracts (6 and 12 months) are associated with higher APRs than longer term contracts (18 and 24 months). Arguably, the 90 days same as cash promotion period represents a significant portion of the duration of shorter contracts, so its impact is large. Since we set $\pi = m_S + M_T$–the minimum reasonable value, early purchases are relatively cheap for early purchasers if performed within three months, which raises the financing cost for pay to term customers under a zero profit condition. Indeed, Table 8 reveals that early purchases do happen with a relatively higher frequency for short term contracts. Another explanation is that, ceteris paribus, the impact of the delivery and set-up costs on the periodic payment is all the more significant as the length of the contract is short, which raises the APR.

We also examine the revenue decomposition per outcome. This allows us to identify and quantify the core line of RTO business: means of acquisition versus rentals. Recall that the total value of the contract is $V_N + p_N - \lambda m_S v_0 - m_S v_0$, which, at the equilibrium, must be equal to v_0. Total revenue v_0 can be decomposed as follows:

$$v_0 = V_N^M + V_N^L + V_N^H,$$

where

$$\begin{aligned} V_N^M &= \overline{q}_{M,N}[\frac{1-\beta^{N+1}}{1-\beta}(p_N - \lambda m_S v_0) - m_S v_0] \\ V_N^L &= \sum_{k=1}^{N} \overline{q}_{L,k}[\beta^k[(1-\psi)v_k - m_S v_0] + \frac{1-\beta^k}{1-\beta}(p_N - \lambda m_S v_0) - m_S v_0] \\ V_N^H &= \sum_{k=1}^{N} \overline{q}_{H,k}[\beta^k D_k + \frac{1-\beta^k}{1-\beta}(p_N - \lambda m_S v_0) - m_S v_0]. \end{aligned}$$

The net revenue generated by an ongoing contract at the end of period $k-1$ (beginning of period k) is $\sum_{s=0}^{k-1} \beta^s (p_N - \lambda m_S v_0) - m_S v_0$, which is equal to $\frac{1-\beta^k}{1-\beta}(p_N - \lambda m_S v_0) - m_S v_0$; thus, it is easy to see that V_N^M, V_N^L and V_N^H are respectively the (expected) revenues generated by pay to term, return and early purchase customers.

We quantitatively illustrate these findings for our preferred parameter values[15] of $\delta_A = 0.3$

[15] Results with other parameter values of δ_A and m_S are qualitatively similar.

and $m_S = 0.15$; see Table 11 for the results.

Table 11: Revenue Decomposition per Outcome (in Percentages)

	Weekly				Monthly			
	6 m	12 m	18 m	24 m	6 m	12 m	18 m	24 m
Pay to Term	0.247	0.14	0.17	0.178	0.498	0.234	0.233	0.175
Early Purchase	0.259	0.14	0.088	0.039	0.213	0.302	0.27	0.157
Return and Default	0.494	0.72	0.742	0.783	0.289	0.464	0.497	0.668
All (v_0)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 11 displays the revenue decomposition per outcome for $\delta_A = 0.30$ and $m_S = 0.15$.

Table 11 reveals that revenues generated by purchases are significantly higher for very short-term contracts. As the length of the contract rises, revenues generated by returns increase, accounting for nearly three quarters of the total revenue for weekly payment schedules and up to two third of the total revenue for monthly payment schedules. Overall, revenues generated by purchases are significantly higher under monthly, as compared to weekly, payment schedules.

The RTO industry has received much regulatory attention and various consumer protection group have targeted the RTO sector by endorsing specific regulations and conducting awareness campaign. A common perception among many is that, similar to the payday industry, RTO financing is usurious and predatory. In this paper, despite an earnest attempt, we are unable to rationalize the high observed markups and APRs associated with RTO transactions at our preferred parameter values of $m_S = 0.15$ and $\delta_A = 0.3$. Our model does ignore certain other, albeit minor, special features and promotions of an RTO agreement. For example, it does not allow for the reinstatement option and special promotions like "rent today with no payment for a month", product upgrade, etc. The assumption that the service cost is proportional to the retail value may not always be accurate. For example, the delivery and setup costs of an inexpensive printer may be more that what the model assumes. Also, our model ignores the cost of processing returned items such as inventorying, cleaning, possibly refurbishing, etc. Another feasible explanation is market imperfections. Barriers to entry may exist as traditional retail stores are reluctant to enter the RTO market, fearing that the perceived reputation of RTO stores, as well as a change in consumer behavior from a straight purchase to a rent-to-own, can dampen their retail business.

To investigate the extent of the lack of perfect competition implied by our model, we compute the implied profit rate ξ_N on a contract of length N in order to match the observed APR values reported in table 4. Equation 2 is adjusted by replacing its right hand side by $(1 + \xi_N)v_0$. We use our preferred parameter values of $m_S = 0.15$ and $\delta_A = 0.3$. Furthermore, we use the actual frequency of weekly and monthly payment schedules, assuming equal implied profit rates for both schedules, to evaluate aggregate implied profit rates. For $N = 6$, 12, 18 and 24 months, we find that the implied profit rates are equal to 10.8%, 13.0% , 17.9% and 11.2% respectively. The implied profit rates seem fairly consistent across contract lengths; furthermore, in our opinion, the magnitude does not seem outrageous.

6 Conclusion

The rent-to-own industry, along with others engaged in so-called subprime financing, has received much regulatory scrutiny. This is due, in part, to the very high costs faced by RTO customers. There is also the concern that the low income, possibly unsophisticated, consumers who form the core of the customer base could easily be exploited. In this paper, we develop a rational-expectations competitive equilibrium model to explore the financing costs of an RTO contract incorporating the features that make it differ starkly from a regular credit purchase. Our model departs from standard option frameworks since, unlike financial securities, RTO products are illiquid, indivisible and a customer's decision to exercise an option is often dictated by idiosyncratic shocks. Using detailed transactional data, we infer how customers exercise the embedded options to determine markups and APRs predicted by our model for several product categories, contractual lengths and payment schedules. We also utilize our transactional data to directly calculate observed markups and APRs.

Our model matches the patterns displayed by the observed markups and APRs very well; however, it falls a little short in matching the levels for reasonable parameter values. We argue that market imperfections is a possible explanation for the gap between the observed and predicted markups and APRs. There may be barriers to entry as traditional retail stores are reluctant to enter the RTO market, fearing that the perceived reputation of RTO stores, as well as a change in consumer behavior from a straight purchase to a rent-to-own, can dampen their retail business.

Although the financing costs incurred by pay to term customers is the primary focus in much of the RTO literature, our transactional data show that these customers represent only about 15 percent of the population and, for the most part, generate less than a quarter of the total revenue for the RTO store. Another 15 percent are early purchase customers for whom an RTO agreement is a convenient way to experiment with a product before (possibly) acquiring it. Most of the clientele consists of return customers especially for contracts under weekly payment schedule. Our calibrations show that RTO is an expensive service to offer, especially with additional pick-up and restocking costs of returns, which is passed on to all RTO customers. However, an RTO agreement is attractive to customers, with well-defined rental needs, given the few expensive alternatives available. In fact, rentals account at least half of the total revenue for an RTO store. In sum, free services coupled with the high incidence of return and early purchase, especially in the early stages of the contract, raises the financing costs of a truly financially constrained, pay to term, customer for whom an RTO may be the only feasible way of acquisition.

To the best of our knowledge, this paper is among the very first attempts to offer a unified theoretical setting for the RTO business and to explore its implications by relying on calibrations obtained from a unique database of RTO transactions. Overall, the paper contributes to the public policy debate on subprime lending as well as to the economic literature by deepening our understanding of the explicit sources and magnitudes of the costs of RTO arrangements to consumers.

7 References

Abell Foundation, 2012. Rent-to-own: exploiting baltimore's poors 25, 1-12.

Agarwal, S., Skila, P. M., Tobacman, J., 2009. Payday loans and credit cards: new liquidity and credit puzzles? American Economic Review 99, 412-417.

Anderson, M. H., Jackson, R.,. 2001. A reconsideration of rent-to-own. The Journal of Consumers Affairs 35, 295-306.

Anderson, M. H., Jaggia, S.,. 2009. Rent-to-own agreements: customers characteristics and contract outcomes. Journal of Economics and Business 61, 51-69.

Anderson, M. H., Jaggia, S., 2012. Return, purchase and skip? Outcome, duration and consumer behavior in the rent-to-own market. Empirical Economics 43, 313-334.

Association of Progressive Rental Organizations (APRO), 2011. APRO Data Survey. Austin, TX.

Beales, H. J., Eisenach, J. A., Litan, R. E., 2012. Consumer welfare implications of regulating rent-to-own transactions. http://papers.ssrn.com/sol3 /papers.cfm?abstract_id=2060984.

Czervonko, A., 2012. An analysis of the rent-to-own industry. http://www.columbia.edu/~aec2156.

Bureau of Economic Analysis, 2007. Table A. BEA rates of depreciation, service lives, declining balance rates. United States Department of Commerce, Washington, DC.

Doms, M. E., Dunn, W. E., Oliner, S. D., Sichel, D. E., 2004. How fast computers and do personal computers depreciate? Concepts and new estimates. In: Poterba, J. M., (Ed.). Tax Policy and the Economy 18, MIT Press, Boston: Massachusetts, 37-80

Federal Trade Commission (FTC), 2000. Survey of rent-to-own customers. Bureau of Economic Staff Report, Washington, DC.

Grenadier, S. R., 1995. Valuing lease contracts: a real-options approach. Journal of Financial Economics 38, 297-331.

Gross, D. B., Souleles, N. S., 2002. Do liquidity constraints and interest rates matter for consumer behavior? Evidence from credit card data. Quarterly Journal of Economics 117, 149-185.

Hill, R. P., Ramp, D. L., Silver, L., 1998. The rent-to-own industry and pricing disclosure tactics. Journal of Public Policy and Marketing 17, 3-10.

Jappelli, T., 1990. Who is credit constrained in the US economy? Quarterly Journal of Economics 105, 219-234.

Laurence, E. C., Elliehaussen, G., 2008 A comparative analysis of payday loan customers. Contemporary Economics Policy 26, 299-316.

Lacko, J. M., McKernan, S. M., Hastak, M., 2002. Customer experience with rent-to-own transactions. Journal of Public Policy and Marketing 21, 126-138.

Lacko, J. M., McKernan, S. M., Hastak, M., 2003. Empirical evidence on the determinants of rent-to-own use and purchase behavior. Economic Development Quarterly 17, 33-52.

McConnell, J., Schallheim, J.S., 1983. Valuation of asset lease contracts.Journal of Financial Economics 12, 237-261.

Morse, A., 2011. Payday lenders: heroes or villains? Journal of Financial Economics 102, 28-44.

Rothschild, M., Stiglitz, J. E., 1976. Equilibrium in competitive insurance markets: an essay on the economics of imperfect information. Quarterly Journal of Economics 90, 629-649.

Swagler, R. M., Burton, J., Koonce Lewis, J., 1995. The alternative financial sector: an overview. Advancing the Consumer Interest 7, 7-12.

Swagler, R. M., Wheeler, P., 1989. Rental-purchase agreements: a preliminary investigation of consumer attitudes and behaviors. The Journal of Consumer Affairs 23, 145-160.

United States Census Bureau, 2010. Annual retail trade report, United States Department of Commerce, Washington, DC.

Walden, M., 1990. The economics of rent-to-own contracts.The Journal of Consumer Affairs 24, 326-337.

Winn, Edward L. III. 2011. "Rent-to-Own at the Federal Level." Association of Progressive Rental Organizations (APRO), Austin, TX.

Zikmund-Fisher, B., Parker, A., 1999. Demand for rent-to-own contracts: a behavioral economic explanation. Journal of Economic Behavior and Organization 38, 199-216.

8 Appendix

A1. Proof of the expression of p_N under iid outcomes. We start with the following lemma.

Lemma 1 $\sum_{k=1}^{N}(\beta q_M)^{k-1}(N+1-k) = \frac{(\beta q_M)^{N+1}-(N+1)\beta q_M+N}{(1-\beta q_M)^2}$.

Proof: Note that $\sum_{k=1}^{N}(\beta q_M)^{k-1}(N+1-k) = (\beta q_M)^N \sum_{j=0}^{N} j(\beta q_M)^{-j}$ $(j = N+1-k)$. Define auxiliary function $F(x) = \sum_{k=0}^{K} e^{kx} = \frac{e^{(K+1)x}-1}{e^x-1}$, so that $F'(x) = \sum_{k=0}^{K} ke^{kx}$, and therefore $\sum_{k=0}^{K} ke^{kx} = \frac{Ke^{(K+2)x}-(K+1)e^{(K+1)x}+e^x}{(e^x-1)^2}$. For $e^x = (\beta q_M)^{-1}$, we find that

$$(\beta q_M)^N \sum_{k=0}^{N} k(\beta q_M)^{-k} = \frac{(\beta q_M)^{N+1}-(N+1)\beta q_M+N}{(1-\beta q_M)^2} = J_N(\beta q_M). \blacksquare$$

Using relationship (1), we find that

$$\begin{aligned}\sum_{k=1}^{N}(\beta q_M)^{k-1}D_k &= (1+\pi)\frac{1-(\beta q_M)^T}{1-\beta q_M}v_0 + (1-\varpi)(\beta q_M)^{N-1}p_N + \varpi\frac{(\beta q_M)^{N+1}-(N+1)\beta q_M+N}{(1-\beta q_M)^2}p_N \\ &\quad -\varpi(N+1)\frac{1-(\beta q_M)^T}{1-\beta q_M}p_N - (1-\varpi)\frac{T(\beta q_M)^{T+1}-(T+1)(\beta q_M)^T+1}{(1-\beta q_M)^2}p_N. \blacksquare\end{aligned}$$

A2. Proof of Property 1. Set $\beta q_M = e^{-x}$, with $x > 0$ so that

$$p_N = \frac{L(x)}{D(x)},$$

where $L(x) = 1 + m_S - (1-\psi)\frac{q_L}{q_M}\sum_{k=1}^{N} e^{-kx}\left[(1-\delta)^k - m_S\right] - (1+\pi)\frac{q_H}{q_M}\sum_{k=1}^{T} e^{-kx}$ and $D(x) = 1 + \sum_{k=1}^{N} e^{-kx} + \frac{q_H}{q_M}\sum_{k=T+1}^{N} e^{-kx}\varpi_{k,N} - \frac{q_H}{q_M}\sum_{k=1}^{T} ke^{-kx}$. By assumption A.1, for all $k \leq N$, we have $(1-\delta)^k - m_S \geq 0$. Clearly L is increasing in x whereas D is decreasing in x if either $\frac{T}{N}$ or $\frac{q_H}{q_M}$ are sufficiently small, so $\frac{\partial p_N}{\partial x} > 0$, and $\frac{\partial p_N}{\partial \beta} = \frac{\partial p_N}{\partial x} \times \frac{\partial x}{\partial \beta} < 0$ as $\frac{\partial x}{\partial \beta} = -\frac{1}{\beta} < 0$. Next, observe that

$$p_N = \frac{1 + m_S\left[1 + \sum_{k=1}^{N}\beta^k \overline{q}_{L,k} + \lambda(1 + \sum_{k=1}^{N}\beta^k\overline{q}_{M,k})\right] - (1-\psi)\sum_{k=1}^{N}\left[(\beta(1-\delta))^k \overline{q}_{L,k}\right] - (1+\pi)\sum_{k=1}^{T}\beta^k\overline{q}_{H,k}}{1 - \sum_{k=1}^{T}\left[\beta^k\overline{q}_{H,k}k\right] + \sum_{k=1}^{N}\beta^k\overline{q}_{M,k} + \sum_{k=T+1}^{N}\left[\beta^k\overline{q}_{H,k}\varpi_{k,N}\right]} v_0.$$

Set $a_k = \beta^k\left[(1-\psi)\left[(1-\delta)^k - m_S\right]\right]\overline{q}_{H,k} > 0$, $b_{k,N} = \lambda_k + \mu_{k,N}(N+1-k)$, with $\lambda_k = \beta^k\overline{q}_{M,k} > 0$, $\mu_{k,N} = \varpi\beta^k\overline{q}_{H,k} > 0$ if $k < N$, $\mu_{N,N} = \beta^N\overline{q}_{H,N} > 0$ and $A_N = 1-(1+\pi)\sum_{k=1}^{T}\left[\beta^k\overline{q}_{H,k}\right] + \sum_{k=1}^{N} a_k > 0$ and $B_N = 1 + \sum_{k=1}^{T}\left[\beta^k(\overline{q}_{M,k} - k\overline{q}_{H,k})\right] + \sum_{k=T+1}^{N} b_{k,N} > 0$. Then, for $N \geq T+2$, observe that $B_{N+1} = B_N + \Delta_{N+1}$, with $\Delta_{N+1} = \lambda_{N+1} + \sum_{k=T+1}^{N-1}\mu_{k,N+1} + \mu_{N+1,N+1} + (2\varpi - 1)\mu_{N,N} > 0$ as $\varpi > 0.5$. Then, we have $p_N = \frac{1+m_s-A_N}{B_N}$ and it follows that

$$p_{N+1} - p_N = -\frac{p_N\Delta_{N+1} + a_{N+1}}{B_{N+1}} < 0.$$

The other properties are straightforward to establish. ■

A3. Proof of Property 2. Since $\frac{N+1}{\theta_N} = \sum_{k=0}^{N}\frac{1}{(1+r_N)^k}$, clearly, as θ_N increases, then r_N increases too. Define the auxiliary function $G_N(y) = (N+1)\left(\sum_{k=0}^{N}\frac{1}{y^k}\right)^{-1} = \frac{y^N(y-1)}{y^{N+1}-1}$. Note that $G_N(1 + r_N) = \theta_N$, so that $\frac{\partial^2 r_N}{\partial\theta_N^2} = -\frac{G''_N(1+r_N)}{\left[G'_N(1+r_N)\right]^3}$. Since $G'_N > 0$, it is enough to show that $G''_N < 0$ to establish that r_N is convex in θ_N. It follows that

$$G'_N(y) = \frac{y^{2N} - (N+1)y^N + Ny^{N-1}}{(y^{N+1}-1)^2}$$

$$G''_N(y) = -\frac{y^{N-2}\left[2y^{2(N+1)} + \left[(N-\frac{1}{2})^2 - \frac{3}{4}\right]y^{N+2} + Ny^{N-1}[N(2y-1) + 5y - 1] + N(N-1)\right]}{(y^{N+1}-1)^3}.$$

Clearly, for $N \geq 2$ and $y > 1$ (so that $r_N > 0$), we have $G_N'' < 0$. Finally, recall that $v_0 = p_N \frac{1+r_N}{r_N}\left[1 - \frac{1}{(1+r_N)^{N+1}}\right]$; differentiating both sides of the equality with respect to N yields

$$\begin{aligned} 0 &= \frac{1+r_N}{r_N}\left[1 - \frac{1}{(1+r_N)^{N+1}}\right]\frac{\partial p_N}{\partial N} + p_N \frac{1+r_N}{r_N}\frac{\ln(1+r_N)}{(1+r_N)^{N+1}} \\ &\quad - \frac{\partial r_N}{\partial N}\sum_{k=0}^{N}\frac{k p_N}{(1+r_N)^{k+1}}, \end{aligned}$$

which implies that

$$\frac{\partial r_N}{\partial N}\sum_{k=0}^{N}\frac{k}{(1+r_N)^{k+1}} = -\frac{1+r_N}{N r_N}\frac{(1+r_N)^{N+1}-1}{(1+r_N)^{N+1}}\left[\varepsilon_{p_N} - \frac{N\ln(1+r_N)}{(1+r_N)^{N+1}-1}\right],$$

where $\varepsilon_{p_N} = -\frac{\partial p_N}{\partial N} \times \frac{N}{p_N}$. ■